RESOLUÇÃO DA PROVA DE MATEMÁTICA = UFRGS 2024
Prof. Jairo

46) A e B

$$\left[\left(1+\frac{1}{2}\right)\cdot\left(1+\frac{1}{2}\right)\cdot\left(1+\frac{1}{2}\right)\cdot\ldots\cdot\left(1+\frac{1}{2}\right)\right]^2 =$$

$$=\left(\frac{3}{2}\cdot\frac{4}{3}\cdot\frac{5}{4}\cdot\frac{6}{5}\cdot\ldots\cdot\frac{99}{98}\cdot\frac{100}{99}\right)^2=\left(\frac{100}{2}\right)^2=$$

$$=50^2=2500$$

O número 2500 é múltiplo de 4.

(quando a última dezena é "00" ou é divisível por 4).

O número 2500 é múltiplo de 5.

(quando o algarismo das unidades é "0" ou "5".

Portanto, tem-se duas alternativas corretas (A e B).

Questão para anulação.

47) D

I)

$$1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}\ldots > 2+\frac{2}{3}+\frac{2}{9}+\frac{2}{27}\ldots$$

PG infinita de razão $\frac{1}{2}$ — PG infinita de razão $\frac{1}{3}$

$$S_1=\frac{a_1}{1-q} \qquad S_2=\frac{a_1}{1-q}$$

$$S_1=\frac{1}{1-\frac{1}{2}}=\frac{1}{\frac{1}{2}} \qquad S_2=\frac{2}{1-\frac{1}{3}}=\frac{2}{\frac{2}{3}}$$

$$S_1=2 \qquad S_2=3$$

$$2>3 \quad (F)$$

II

$$\sqrt{7+10}<\sqrt{7}+\sqrt{10}$$

$$\sqrt{17}<\sqrt{7}+\sqrt{10}$$

$$4<\sqrt{17}<5 \qquad 2<\sqrt{7}<3 \qquad 3<\sqrt{10}<4$$

$$\sqrt{17}<\sqrt{7}+\sqrt{10}$$

$$4,\_\_<2,\_\_+3,\_\_ \quad (V)$$

$III)$

$$6.5^{10}<5.6^{10}$$

$$\frac{5^{10}}{5}<\frac{6^{10}}{6}$$

$$5^9<6^9 \quad (V)$$

48) D

$$x^2 + 2x - 15 = 0$$

$$Soma\ das\ raízes = a + b = -2$$

$$\text{Pr}oduto\ das\ raízes = a.b = -15$$

$$\left(ab\right)^{a+b} = (-15)^{-2} = \left(\frac{1}{15}\right)^2 = \frac{1}{225}\ (D)$$

49) C

$$I)\ \ (V)$$

$$Z = 3 + 4i$$

$$\left|Z\right| = \sqrt{a^2 + b^2} = \sqrt{3^2 + 4^2} = \sqrt{25} \quad \left|Z\right| = 5$$

$$II)\ (V)$$

$$|u \cdot v| = |u| \cdot |v|$$

$$u \cdot v = \left(1 + i\right) \cdot \left(1 - i\right) = 1^2 - i^2 = 1 - (-1) = 2$$

$$|u \cdot v| = \sqrt{a^2 + b^2} = \sqrt{2^2 + 0^2} = \sqrt{4} = 2$$

$$|u| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$|v| = \sqrt{1^2 + (-1)^2} = \sqrt{1+1} = \sqrt{2}$$

$$|u \cdot v| = |u| \cdot |v|$$

$$2 = \sqrt{2} \cdot \sqrt{2}$$

$$III)\ \ (F)$$

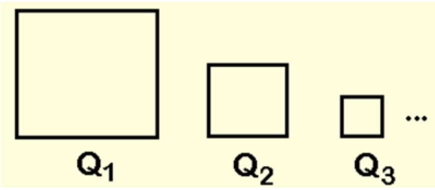
$$w = (x-3) + (x+4)i \quad Para\ ser\ real\ a\ parte\ imaginária\ é\ nula.$$

$$Então,\ x + 4 = 0 \quad Se\ x = -4, w\ é\ um\ número\ real.$$

$$Se\ x = 3,\ w = \left(3-3\right) + (3+4)i \quad w = 7i.$$

$$Por\tan to, se\ x = 3,\ w\ não\ é\ um\ número\ real.$$

50) B



$$\ell_1 = \log(2) \qquad \ell_2 = \log\left(\sqrt{2}\right) \qquad \ell_3 = \log\left(\sqrt[4]{2}\right)$$

$$A_1 = (\log 2)^2 \quad A_2 = \left(\log\sqrt{2}\right)^2 \quad A_3 = \left(\log\sqrt[4]{2}\right)^2$$

$$razão\, da\, PG \;\; q = \frac{\left(\log\sqrt{2}\right)^2}{(\log 2)^2} = \left(\frac{\log\sqrt{2}}{\log 2}\right)^2$$

Pelo conceito de mudança de base:

$$\left(\frac{\log\sqrt{2}}{\log 2}\right)^2 = \left(\log_2\sqrt{2}\right)^2 = \left(\log_2 2^{1/2}\right)^2 = \left(\frac{1}{2}\right)^2$$

$$q = \frac{1}{4}$$

Soma dos inf *initos termos* (inf *initas áreas*)

$$Limite\ da\ Soma = \frac{a_1}{1-q} = \frac{(\log 2)^2}{1-\frac{1}{4}} = \frac{(\log 2)^2}{\frac{3}{4}} =$$

$$= (\log 2)^2 \cdot \frac{4}{3}$$

51) B

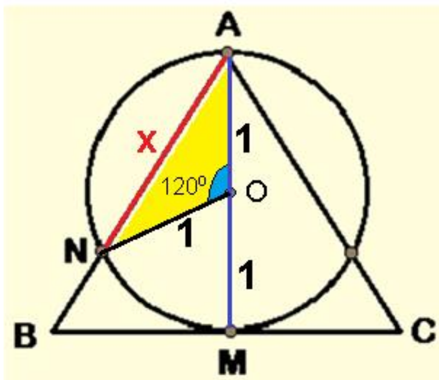
$$h_\Delta = \frac{\ell\sqrt{3}}{2} = \frac{4\sqrt{3}}{3} \times \frac{\sqrt{3}}{2} = \frac{12}{6} \qquad h_\Delta = 2$$

Lei dos Cos *senos no* Δ_{AON} :

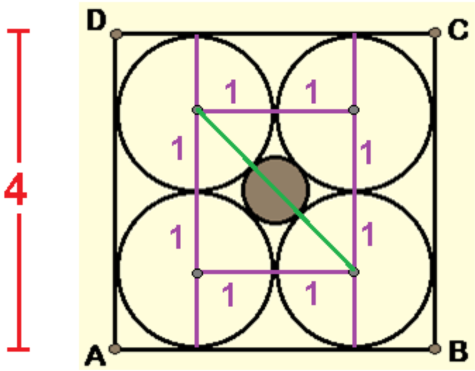
$$x^2 = 1^2 + 1^2 - 2.1.1.\cos 120^o$$

$$x^2 = 1 + 1 - 2.1.1.\frac{1}{2} \qquad x^2 = 1+1+1$$

$$x^2 = 3 \qquad x = \sqrt{3}$$



52) A



$$Diagonal\ do\ quadrado:$$

$$d = \ell\sqrt{2} \qquad d = 2\sqrt{2}$$

$$diâmetro\ do\ círculo\ menor:$$

$$diâm = diagonal - 2r$$

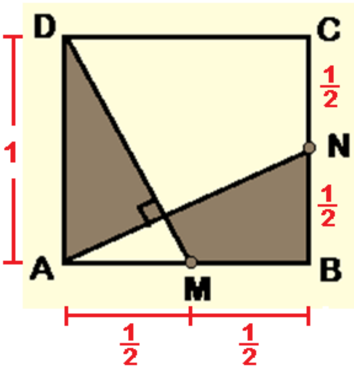
$$diâm = 2\sqrt{2} - 2$$

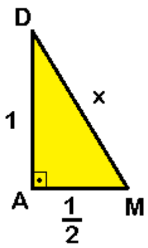
$$diâm = 2(\sqrt{2} - 1) \qquad raio = \sqrt{2} - 1$$

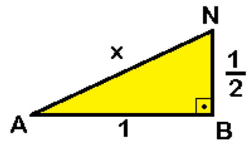
$$A = \pi.r^2 = \pi(\sqrt{2} - 1)^2 = \pi\left((\sqrt{2})^2 - 2.\sqrt{2}.1 + 1^2\right)$$

$$A = \pi\left(2 - 2\sqrt{2} + 1\right) \qquad A = \pi\left(3 - 2\sqrt{2}\right)$$

53) D







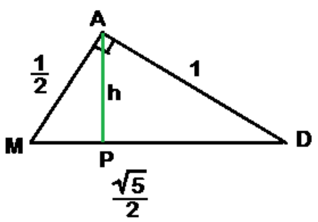
$$Área\,\Delta_{ADM} = \frac{base \times altura}{2} = \frac{\frac{1}{2} \times 1}{2}$$

$$Área\,\Delta_{ADM} = \frac{1}{4} \qquad Área\,\Delta_{ABN} = \frac{1}{4}$$

$$Á_{sombreada} = \frac{1}{4} + \frac{1}{4} - 2A\Delta_{AMP}$$

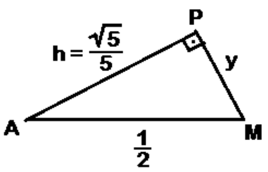
$$HIP^2 = CAT_1^{\,2} + CAT_2^{\,2}$$

$$x^2 = 1^2 + \left(\frac{1}{2}\right)^2 \qquad x^2 = \frac{5}{4} \qquad x = \frac{\sqrt{5}}{2}$$



$$\frac{\sqrt{5}}{2} \cdot h = \frac{1}{2} \cdot 1$$

$$h = \frac{1}{\sqrt{5}} \qquad h = \frac{\sqrt{5}}{5}$$



$$\left(\frac{1}{2}\right)^2 = \left(\frac{\sqrt{5}}{5}\right)^2 + y^2$$

$$\frac{1}{4} - \frac{1}{5} = y^2 \qquad \frac{5-4}{20} = y^2$$

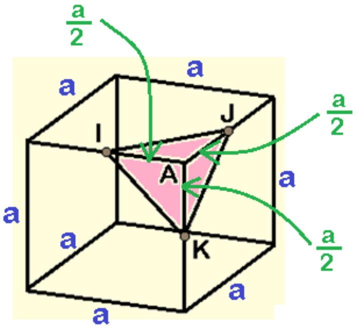
$$y^2 = \frac{1}{20} \qquad y = \frac{1}{2\sqrt{5}} \qquad y = \frac{\sqrt{5}}{10}$$

$$A_{sombreada} = \frac{1}{4} + \frac{1}{4} - 2.\frac{1}{20} = \frac{1}{2} - \frac{1}{10}$$

$$A_{sombreada} = \frac{5}{10} - \frac{1}{10} = \frac{4}{10} = \frac{2}{5}$$

$$A\Delta_{AMP} = \frac{\frac{\sqrt{5}}{5} \cdot \frac{\sqrt{5}}{10}}{2} = \frac{\frac{5}{50}}{2} = \frac{1}{10} \cdot \frac{1}{2} = \frac{1}{20}$$

54) E



$$V_{cubo} = a^3$$

$$V_{pirâmide} = \frac{A_{base} \cdot altura}{3}$$

$$A_{base} = A_{triâng} = \frac{base \cdot altura}{2}$$

$$A_{base} = \frac{\frac{a}{2} \cdot \frac{a}{2}}{2} = \frac{\frac{a^2}{4}}{2} = \frac{a^2}{4} \cdot \frac{1}{2} = \frac{a^2}{8}$$

$$V_{pirâmide} = \frac{\frac{a^2}{8} \cdot \frac{a}{2}}{3} = \frac{\frac{a^3}{16}}{3} = \frac{a^3}{16} \cdot \frac{1}{3} = \frac{a^3}{48}$$

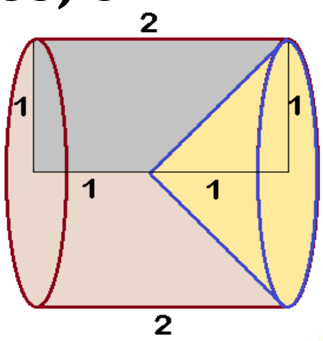
$$V_{8\,pirâmides} = 8 \times \frac{a^3}{48} = \frac{a^3}{6}$$

$$V_{restante} = a^3 - \frac{a^3}{6} = \frac{6a^3 - a^3}{6} = \frac{5a^3}{6}$$

$$A_{sombreada} = \frac{1}{4} + \frac{1}{4} - 2 \cdot \frac{1}{20} = \frac{1}{2} - \frac{1}{10}$$

$$A_{sombreada} = \frac{5}{10} - \frac{1}{10} = \frac{4}{10} = \frac{2}{5}$$

55) C



$$V_{cilindro} = A_{base} \times altura \quad V_{cilindro} = \pi . R^2 . H$$

$$V_{cilindro} = \pi . 1^2 . 2 \quad V_{cilindro} = 2\pi$$

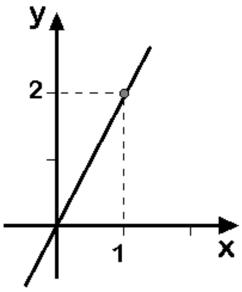
$$V_{cone} = \frac{A_{base} \times altura}{3} \quad V_{cone} = \frac{\pi . R^2 . h}{3}$$

$$V_{cone} = \frac{\pi . 1^2 . 1}{3} \quad V_{cone} = \frac{\pi}{3}$$

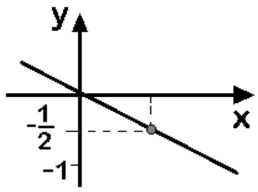
$$V_{sólido} = 2\pi - \frac{\pi}{3} = \frac{6\pi - \pi}{3} \quad V_{sólido} = \frac{5\pi}{3}$$

56) C

$f(x) = 2x$



$g(x) = -\dfrac{x}{2}$

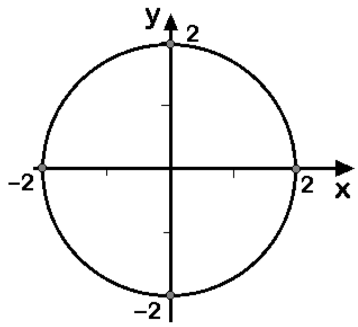


$$h(x) = \sqrt{4 - x^2}$$

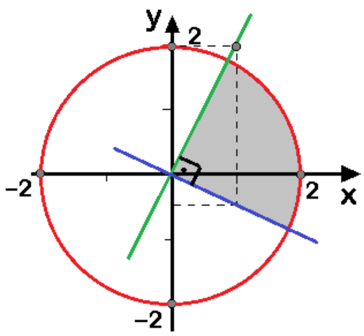
$$h(x) = \sqrt{4 - x^2}$$

$$y^2 = 4 - x^2$$

$$x^2 + y^2 = 4$$



Representando as 3 funções no mesmo sistema cartesiano:
(e observando a área delimitada pelas interseções)

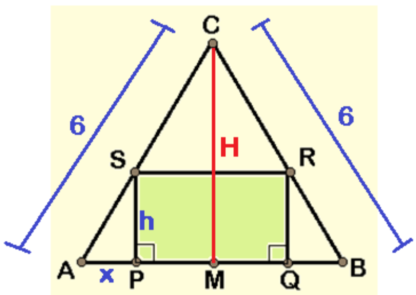


Setor circular de 90^o $\left(\frac{1}{4} \text{ de um círculo} \right)$

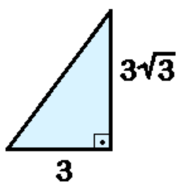
raio $= 2$

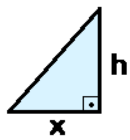
$$A = \frac{\pi r^2}{4} = \frac{\pi 2^2}{4} = \frac{4\pi}{4} \qquad A = \pi$$

57) A



$$H_{\Delta} = \frac{\ell\sqrt{3}}{2} = \frac{6\sqrt{3}}{2} \qquad H = 3\sqrt{3}$$





$$\frac{h}{3\sqrt{3}} = \frac{x}{3} \qquad 3h = x.3\sqrt{3} \qquad h = \frac{x.3\sqrt{3}}{3} \qquad h = \sqrt{3}x$$

$$A_{retângulo} = base \times altura$$

$$A_{retângulo} = (6 - 2x).\sqrt{3}x$$

58) A

I) (V)

PULMÃO = 15342

MAMA = 85620

20% equivale $\frac{1}{5}$ $\quad 15342 \times 5 = 76710 \quad 76710 < 85620$

II) (F)

BEXIGA = 9127

PRÓSTATA = 84992

10% equivale $\frac{1}{10}$ $\quad 9127 \times 10 = 91270 \quad 91270 > 84992$

III) (F)

PULMÃO(h) = 19169

PULMÃO(m) = 15342

10% de 15342 $= 1534,2$

30% de 15342 $= 1534,2 \times 3 = 4602,6$

$Somando: 15342 + 4602,6 = 19944,6$

$19169 < 19944,6$

59) E

$$20 \begin{cases} \text{Antônia-Maria-Francisca} = \text{goleiras (3)} \\ \qquad\qquad\qquad\qquad\qquad\qquad \text{linha (17)} \end{cases}$$

$\boxed{G} \quad \boxed{L_1}\boxed{L_2}\boxed{L_3}\boxed{L_4}$

3 *possib.* $\quad$ Escolher 4 de 17:

$C_{3,1} \qquad C_{17,3}$

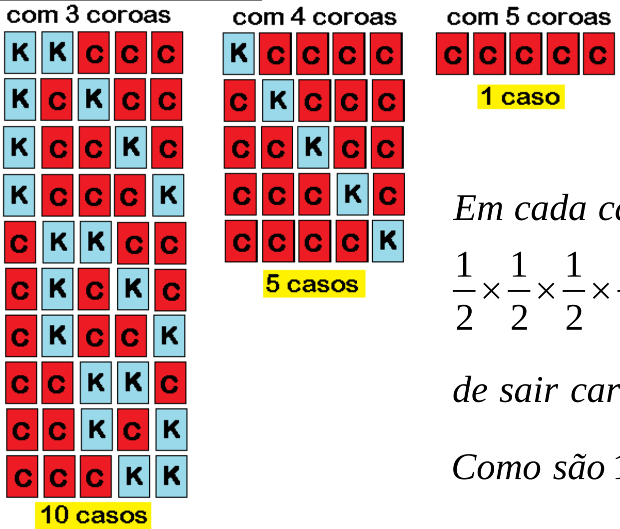
$C_{3,1} + C_{17,3}$ ou $C_{3,1} \times C_{17,3}$?

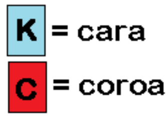
$C_{3,1} \times C_{17,3}$, pois deseja-se a escolha simultânea de uma goleira e de 4 jogadoras de linha. (conetivo "e")

60) E

Observar pelo menos 3 caras em 5 lançamentos de uma moeda não viciada.

Método Intuitivo:





Em cada caso tem-se:

$$\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{32}$$ *(pois tanto a probabilidade de sair cara quanto de sair coroa é a mesma:* $\frac{1}{2}$*)*

Como são 16 *casos* $(10+5+1)$:

$$P(E)=16\times\frac{1}{32}=\frac{1}{2}$$

Utilizando Probabilidade Binomial:

$$P(k)=\binom{n}{k}.p^{k}.q^{n-k}$$

$$P(3)=\binom{5}{3}.\left(\frac{1}{2}\right)^{3}.\left(\frac{1}{2}\right)^{2}=10\cdot\frac{1}{8}\cdot\frac{1}{4}=10\cdot\frac{1}{32}=\frac{10}{32}$$

$$P(4)=\binom{5}{4}.\left(\frac{1}{2}\right)^{4}.\left(\frac{1}{2}\right)^{1}=5\cdot\frac{1}{16}\cdot\frac{1}{2}=5\cdot\frac{1}{32}=\frac{5}{32}$$

$$P(5)=\binom{5}{5}.\left(\frac{1}{2}\right)^{5}.\left(\frac{1}{2}\right)^{0}=1\cdot\frac{1}{32}\cdot1=1\cdot\frac{1}{32}=\frac{1}{32}$$

Somando as probabilidades binomiais:

$$\frac{10}{32}+\frac{5}{32}+\frac{1}{32}=\frac{16}{32}=\frac{1}{2}$$